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SHORT COMMUNICATION

MODIFICATION OF THE SMAC METHOD IN TWO DIMENSIONS

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The Marker-and-Cell (MAC) method, proposed by Harlow and Welch,[1] is an efficient numerical solution technique for investigating the dynamics of an incompressible fluid. However, the method requires the solution of the Poisson equation for pressure distribution. This disadvantage is removed by the Simplified MAC (SMAC)[2] technique in which the pressure need not be calculated.

The basic idea is to separate each calculation cycle into two parts

1. The tentative velocity field $\tilde{w}$ is calculated by using an arbitrary pressure field. Correct velocity boundary conditions ensure that this tentative velocity field reproduces the vorticity in a correct way.[3] However, the mass concentration equation is not satisfied.
2. The potential function ϕ is introduced so that the final velocity determined by $w=\tilde{w}+\text{grad}\,\phi$ preserves the vorticity and satisfies the incompressibility condition. The potential function ϕ is calculated from the Poisson equation for mass conservation.

In two dimensions it is more interesting to use the stream function ψ instead of the potential. Assume the momentum equations without the pressure gradient to be

$$\tilde{u}^{n+1}=u^n+\Delta t\left[-u^n\frac{\partial u^n}{\partial x}-v^n\frac{\partial u^n}{\partial y}+\nu\frac{\partial}{\partial y}\left(\frac{\partial u^n}{\partial y}-\frac{\partial v^n}{\partial x}\right)\right] \tag{1}$$

$$\tilde{v}^{n+1}=v^n+\Delta t\left[-u^n\frac{\partial v^n}{\partial x}-v^n\frac{\partial v^n}{\partial y}-\nu\frac{\partial}{\partial x}\left(\frac{\partial u^n}{\partial y}-\frac{\partial v^n}{\partial x}\right)\right]. \tag{2}$$

Since $(\tilde{u}^n, \tilde{v}^n)$ satisfies the continuity equation the vorticity calculated from the equation

$$\omega^{n+1}=\frac{\partial\tilde{v}^{n+1}}{\partial x}-\frac{\partial\tilde{u}^{n+1}}{\partial y} \tag{3}$$

is correct.

The stream function is calculated from the Poisson equation

$$\nabla^2\psi^{n+1}=\omega^{n+1} \tag{4}$$

and the final velocity field has the form

$$u^{n+1}=\partial\psi^{n+1}/\partial y, \qquad v^{n+1}=-\partial\psi^{n+1}/\partial x \tag{5}$$

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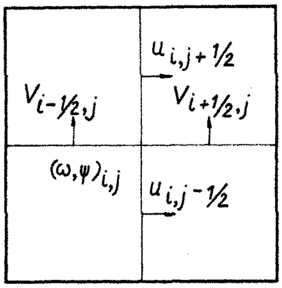

Figure 1. Computational grid and locations of the variables

The method is similar to the ω–ψ method;[4] however, we need not approximate the boundary conditions for vorticity. Contrary to the SMAC method, where the Poisson equation for the potential function had the von Neumann boundary conditions on rigid surfaces, equation (4) assumes the Dirichlet boundary conditions. From equations (1) and (2) it follows that in the stationary state the pressure gradient is given by

$$\partial p/\partial x = \frac{u-\tilde{u}}{\Delta t}, \qquad \partial p/\partial y = \frac{v-\tilde{v}}{\Delta t}.$$

Using the staggered grid (Figure 1) we assume the finite-difference representation of equations (1) and (2) to be in the same form as that for the MAC method.[1] The location ω_{ij} and ψ_{ij} at the points (i, j) leads to the following finite-difference analogue of equations (3), (4) and (5)

$$\omega_{i,j} = (u_{i,j-1/2} - u_{i,j+1/2} + v_{i+1/2,j} - v_{i,j-1/2})/h \tag{3a}$$

$$\psi_{i-1,j} + \psi_{i,j-1} - 4\psi_{i,j} + \psi_{i+1,j} + \psi_{i,j+1} = -h^2\omega_{i,j} \tag{4a}$$

$$\begin{aligned} u_{i,j+1/2} &= (\psi_{i,j+1} - \psi_{i,j})/h, \\ v_{i+1/2,j} &= -(\psi_{i+1,j} - \psi_{i,j})/h \end{aligned} \tag{5a}$$

where h is the spatial step.

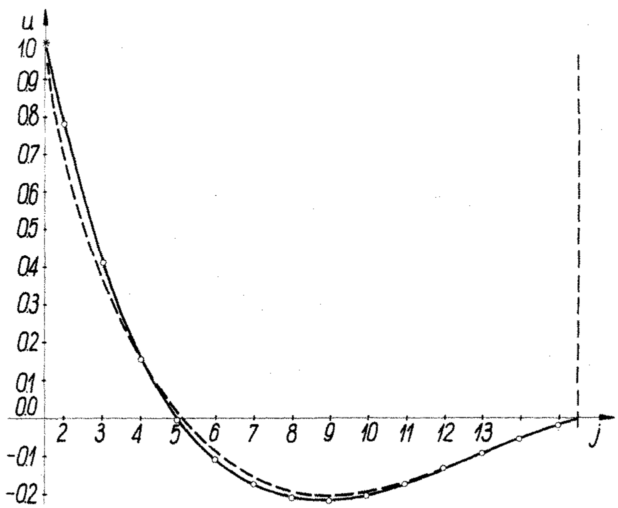

Figure 2. Comparison of two velocity profiles across the vertical central plane of a square cavity with moving wall, Re = 100. (OOO—present calculations, - - - —Burggraf prediction)

To illustrate the method we present the prediction of the velocity profile across the vertical central plane of the square cavity with a moving wall shown in Figure 2. A (14×14) uniform grid was employed, the Reynolds number being 100. The obtained velocity profile is in good agreement with the Burggraf prediction made for (51×51) a uniform grid.[5]

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